

Algebra II
Midterm Exam Review

Name: Key KC

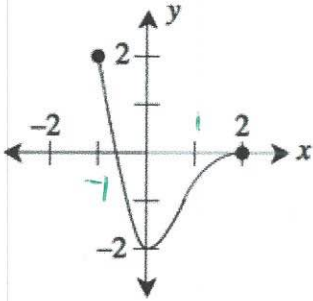
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1. For $f(x) = (x - 4)^2$, evaluate each of the following.

a. $f(3) = (3 - 4)^2$
 $(-1)^2$
1

b. $f(-2) = (-2 - 4)^2$
 $= (-6)^2$
36

2. What is the domain and range of the following function?



$D: [-1, 2]$ or $-1 \leq x \leq 2$

$R: [-2, 2]$ or $-2 \leq y \leq 2$

3. Consider the equation $6x - 4y = 12$.

a. Write the equation in slope-intercept form.

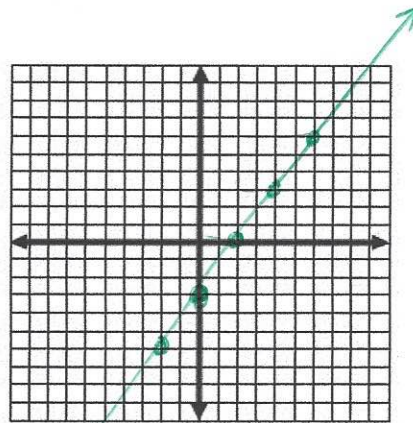
b. Graph the equation.

$y = mx + b$

(Solve for y; there will still be an x in the equation)

$$\begin{array}{r} 6x - 4y = 12 \\ -6x \qquad -6x \\ \hline -4y = -6x + 12 \\ \frac{-4y}{-4} = \frac{-6x + 12}{-4} \end{array}$$

$y = \frac{3}{2}x - 3$



$y = \frac{3}{2}x - 3$
↑ ↑
m b

5. Solve for x: $3(x + 2) - 2(2x - 3) = 13$

$$3x + 6 - 4x + 6 = 13$$

$$\begin{array}{r} -1x + 12 = 13 \\ -12 \quad -12 \\ \hline -1x = 1 \end{array}$$

$$\frac{-1x}{-1} = \frac{1}{-1} \rightarrow \boxed{x = -1}$$

6. If $f(x) = 3x^2 - 8x + 4$, calculate the following:

a. $f(2)$

$$\begin{aligned} 3(2)^2 - 8(2) + 4 \\ 3(4) - 16 + 4 \\ 12 - 16 + 4 \\ \boxed{0} \end{aligned}$$

b. $f(-2)$

$$\begin{aligned} 3(-2)^2 - 8(-2) + 4 \\ 3(4) + 16 + 4 \\ 12 + 16 + 4 \\ \boxed{32} \end{aligned}$$

c. Is f an even, odd or neither function?

Neither. $f(x) \neq f(-x)$
 $f(2) \neq f(-2)$

The function has no symmetry over the y axis or the origin (0,0).

c. If $f(x) = 0$, solve for x .

$$0 = 3x^2 - 8x + 4 \quad \text{factor}$$

$12x^2$	$-2x$
$-16x$	$+4$

x	-2
$3x$	4

$$\rightarrow (x-2)(3x-2) = 0$$

$$x-2=0 \quad \boxed{x=2}$$

$$3x-2=0$$

$$3x=2 \quad \boxed{x=2/3}$$

7. Simplify the expression $6 + 12 \div 2^2 \cdot 3 + 5(3-2)^3$. Show your work clearly.

$$6 + 12 \div 2^2 \cdot 3 + 5(+1)^3$$

$$6 + 12 \div 2^2 \cdot 3 + 5$$

$$6 + 12 \div 4 \cdot 3 + 5$$

$$6 + 3 \cdot 3 + 5$$

$$6 + 9 + 5 = \boxed{20}$$

PEMDAS

L → R L → R

8. Multiple Choice: $(3x)^3 = \underline{\quad}$? Explain your reasoning or show the work that helped you decide on your answer.

A. $27x^3$

B. $9x^3$

C. $3x^3$

D. None of these

$$(3x)^3 = (3x)(3x)(3x)$$

$$27x^3$$

$$y = a \cdot b^{(x-h)} + k$$

exponential

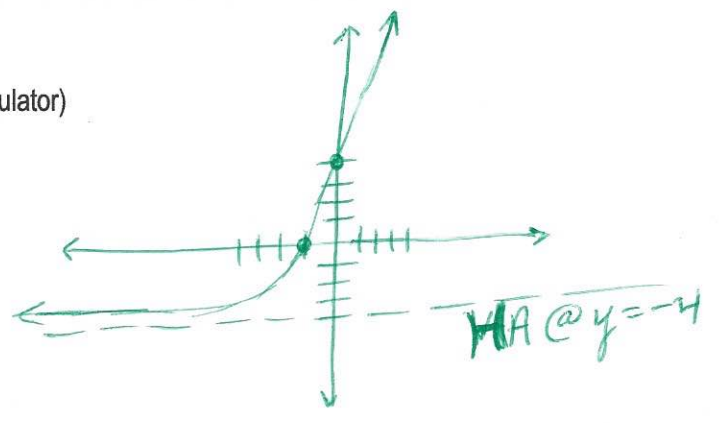
9. Investigate the function $y = 2^{(x+3)} - 4$. (Using your graphing calculator)

x-intercept(s): $(-1, 0)$

y-intercept(s): $(0, 4)$

Asymptote(s): (Hint: Where does the function level off?)

HA $y = -4$



10. Using the equation for a line, $y = mx + b$

a. If $(2, 3)$ and $(4, -1)$ are on the line, write the equation for this line:

linear $y = mx + b$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{4 - 2} = \frac{-4}{2} = -2$$

b. Graph the line.

$$y = mx + b$$

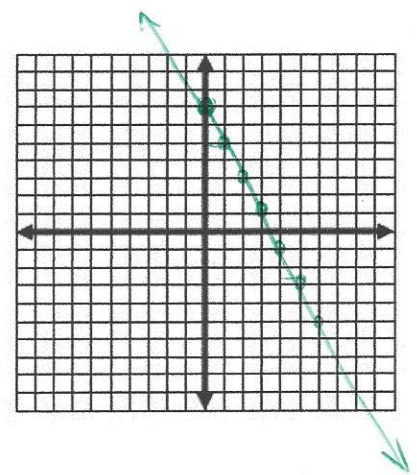
$$3 = (-2)(2) + b$$

$$3 = -4 + b$$

$$7 = b$$

$$y = mx + b$$

$$\rightarrow y = -2x + 7$$



11. Tobey says the equations $3x + 2y = 270$ and $y = \frac{4}{5}x + 20$ intersect at the point $(50, 60)$. Is this true? How do you know? Be clear and complete.

use substitution

$$3x + 2y = 270$$

$$3x + 2\left(\frac{4}{5}x + 20\right) = 270$$

$$3x + \frac{8}{5}x + 40 = 270$$

$$\frac{15x + 8}{5}x + 40 = 270$$

$$\left(\frac{5}{23}\right) \frac{23}{5}x = 230 \left(\frac{5}{23}\right)$$

$$x = 50 \rightarrow$$

$$y = \frac{4}{5}(50) + 20 \rightarrow 40 + 20 = 60$$

Point of intersection $(50, 60)$

OR verify that $(50, 60)$ makes both equations true

$$3x + 2y = 270$$

$$3(50) + 2(60) = 270$$

$$150 + 120 = 270$$

$$270 = 270$$

✓

$$y = \frac{4}{5}x + 20$$

$$60 = \frac{4}{5}(50) + 20$$

$$60 = 40 + 20$$

$$60 = 60$$

✓

yes!

12. Digger the Dog saves the same number of bones each week.

After week 4, he had 23 bones, and after week 8, he had 35 bones. How many did he start with?

Write an equation to model this scenario.

$(4, 23)$ $(8, 35)$
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{35 - 23}{8 - 4} = \frac{12}{4} = 3 = m$

Week(s)	0	1	2	3	4	5	6	7	8
Bone(s)	11	14	17	20	23	26	29	32	35

$y = mx + b$
 $23 = (3)(4) + b$
 $23 = 12 + b$
 $-12 \quad -12$
 $11 = b$

$y = 3x + 11$

13. Find the distance between the following two points using the distance formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(x_1, y_1)
 $(-2, -3)$
 (x_2, y_2)
 $(-4, 4)$

a. (-2, -3) and (-4, 4)

Write answer in simplified radical form (no decimal answer).

$d = \sqrt{(-4 - (-2))^2 + (4 - (-3))^2} = \sqrt{(-4 + 2)^2 + (4 + 3)^2}$
 $= \sqrt{(-2)^2 + (7)^2} = \sqrt{4 + 49} = \sqrt{53}$

Can't simplify

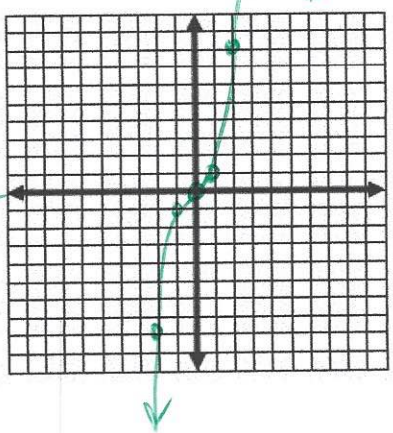
14. Sketch each of the following parent functions:

a. $y = x^3$

Cubic

ODD

x	y
-2	-8
-1	-1
0	0
1	1
2	8

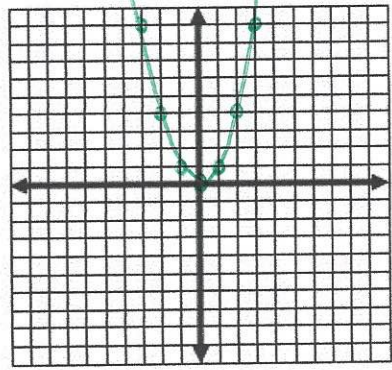


*Determine an equation for the graph translated left 5 and up 2 units.

$y = a(x - h)^3 + k$
 $y = (x + 5)^3 + 2$

b. $y = x^2$

quadratic
even



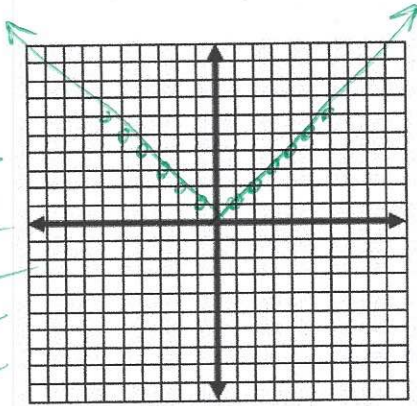
x	y
-2	4
-1	1
0	0
1	1
2	4

*Determine an equation for the graph stretched by 2 and shifted down 1 unit.

$y = a(x - h)^2 + k$
 $y = 2x^2 - 1$

ABSOLUTE VALUE
EVEN

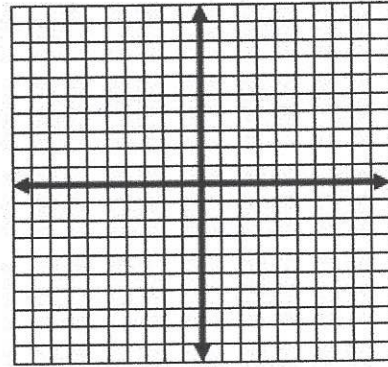
c. $y = |x|$



x	y
-2	2
-1	1
0	0
1	1
2	2

Rational
ODD

d. $y = \frac{1}{x}$



x	y
-2	-1/2
-1	-1
0	UND
1	1
2	1/2

*Determine an equation for the graph flipped, translated right 4 & up 2 units.

$$y = a|x-h| + k$$

$$y = -|x-4| + 2$$

*Determine an equation for the graph translated left 6 and down 4 units.

$$y = a\left(\frac{1}{x-h}\right) + k$$

$$y = \frac{1}{x+6} - 4$$

15. Write the equation of the graph:

$$y = 2(x+1)^2 + 3$$

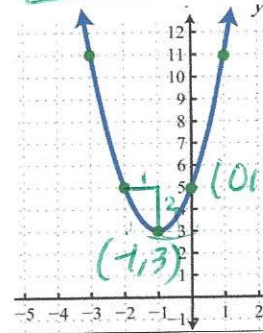
$$y = a(x-h)^2 + k$$

$$5 = a(0 - (-1))^2 + 3$$

$$5 = 1a + 3$$

$$2 = 1a$$

$$a = 2$$



left 1
up 3
vertex = (h, k)
(-1, 3)

16. Cardania bought a car for \$10,000 ten years ago, and it has been depreciating at a rate of 7% per year since she drove it off the showroom floor.

$$y = a \cdot b^x$$

$$1 - 0.07 = 0.93$$

a. Write an equation to model the amount of money the car is worth at time t . What type of function is this?

$$y = 10000(0.93)^t$$
 exponential

b. How much is the car worth today? (may use a calculator)

$$y = 10000(0.93)^{10} = \$4839.82$$

17. Rewrite the following equations in vertex form by completing the square.

a. $y = x^2 + 6x - 3$

$$y + 3 = x^2 + 6x + 9$$

$$y + 12 = (x + 3)^2$$

$$y = (x + 3)^2 - 12$$

b. $y = 2x^2 - 14x + 30$

$$y = 2\left(\frac{7}{2}\right)^2 - 14\left(\frac{7}{2}\right) + 30$$

$$y = 2\left(\frac{49}{4}\right) - (7)(7) + 30$$

$$y = \frac{49}{2} - 49 + 30$$

$$y = \frac{49}{2} - 19 = 24.5 - 19 = 5.5 = \frac{11}{2}$$

$$h = \frac{-b}{2a} = \frac{14}{2(2)} = \frac{14}{4} = \frac{7}{2}$$

Plug in $\frac{7}{2}$ to get k

$$y = 2\left(x - \frac{7}{2}\right)^2 + \frac{11}{2}$$

Slope

$y = mx + b$ (y int start)

18. Sarah was paid \$200 as a bonus for starting a new job. Each week after that she made \$50. She saved everything she made from this job in the bank.

a. Write an equation to represent the amount of money Sarah has in the bank.

x = Weeks
y = \$ in bank

$y = 50x + 200$

b. After being paid for working 15 weeks, how much money does Sarah have saved up?

$y = 50(15) + 200$
 $y = 750 + 200 = \$950$

19. Subtract the following rational expressions by finding a common denominator:

$\frac{4}{x+1} - \frac{2}{x+2} = \frac{4(x+2)}{(x+1)(x+2)} - \frac{2(x+1)}{(x+2)(x+1)} = \frac{4x+8 - 2x-2}{(x+1)(x+2)}$
 $= \frac{2x+6}{(x+1)(x+2)} = \frac{2(x+3)}{(x+1)(x+2)}$

20. Simplify the expression:

$\frac{5x-15}{3x^2+10x-8} \div \frac{x^2+x-12}{3x^2-8x+4} = \frac{5x-15}{3x^2+10x-8} \cdot \frac{3x^2-8x+4}{x^2+x-12}$
 $\frac{5(x-3)}{(3x-2)(x+4)} \cdot \frac{(x-2)(3x-2)}{(x-3)(x+4)} = \frac{5(x-2)}{(x+4)(x+4)}$

$\frac{12x^2-24x}{10x} \times \frac{3x-2}{4(3x^2-2x)}$
 $\frac{12x^2-6x}{-8x} \times \frac{3x-2}{-2(3x^2-10x+4)}$

21. What restrictions, if any, are placed on the value of x in the following expression? (Hint: What can x not equal?)

$\frac{4}{x+1} - \frac{2}{x+2}$

$x+1=0$
 $x \neq -1$

$x+2=0$
 $x \neq -2$

Why can x not equal those values?

Because $x = -1$ and $x = -2$ make the denominators equal 0 & we can't divide by zero. Dividing by zero - the equation is undefined

$$\begin{array}{r} -35x^2 \\ -1x \quad 35x \\ 34x \end{array} \quad \begin{array}{r} x \quad 7 \\ 5x \quad 5x^2 \quad 35x \\ -1 \quad 1x \quad -7 \end{array}$$

22. Simplify the following rational expression: $\frac{5x^2 + 34x - 7}{10x} \cdot \frac{5x}{x^2 + 4x - 21}$

$$\frac{(x+7)(5x-1) \cdot \cancel{5x}}{2 \cdot \cancel{10x} \cdot (x+7)(x-3)} = \frac{(5x-1)}{2(x-3)}$$

23. Erik says $(a + b)^2 = a^2 + 2ab + b^2$, but Henry disagrees. Who is correct? Convince the person who is wrong, why he is wrong and explain what is true.

Eric is correct. $(a+b)(a+b) = a^2 + ab + ab + b^2$
foil or box $\rightarrow a^2 + 2ab + b^2$

24. Simplify the rational expression: $\frac{10x + 25}{2x^2 - x - 15}$

$$\begin{array}{r} -30x^2 \\ 5x \quad -6x \\ -x \end{array}$$

$$\begin{array}{r} x \quad -3 \\ 2x \quad 2x^2 \quad -6x \\ 5 \quad 5x \quad -15 \end{array}$$

$$= \frac{5(2x+5)}{(2x+5)(x-3)} = \frac{5}{x-3}$$

25. Are the following pair of equations or expressions equivalent? Show your work for credit!

a. $(a^2b)^2$ and a^2b^2

$$a^{2 \cdot 2} b^2 = a^4 b^2 \neq a^2 b^2$$

NO

b. $3x - 8y = 16$ and $y = x - 3$

Solve for Y

$$\begin{array}{r} -3x \quad -3x \\ \hline -8y = -3x + 16 \\ \hline -8 \quad -8 \\ \hline y = \frac{3}{8}x - 2 \neq y = x - 3 \end{array}$$

NO

26. Emma has two linear functions: $f(x) = 3x + 1$ and $g(x) = -2x + 5$

a. Find the sum: $f(x) + g(x)$

$$\underbrace{f(x) + g(x)}_{3x+1 + -2x+5}$$

$$x + 6$$

b. Find the difference: $f(x) - g(x)$

$$3x+1 - (-2x+5)$$

$$3x+1 + 2x - 5$$

$$5x - 4$$

c. Find the Product: $f(x) \cdot g(x)$

foil or box

$$(3x+1)(-2x+5)$$

$$-6x^2 + 15x - 2x + 5$$

$$-6x^2 + 13x + 5$$

27. Solve the system of equations: $5x + 3y = 40$
 $y = -x + 8$

$$y = -(8) + 8$$

$$y = 0$$

$$\boxed{\begin{matrix} (8, 0) \\ x \quad y \end{matrix}}$$

$$5x + 3(-x + 8) = 40$$

$$5x - 3x + 24 = 40$$

$$2x + 24 = 40$$

$$2x = 16$$

$$x = 8$$

28. Simplify each of the following.

a. $4^3 \cdot 4^2 = 4^5$ PEMDAS

$$(4 \cdot 4 \cdot 4) \cdot (4 \cdot 4)$$

$$64 \cdot 16$$

$$= \boxed{1024}$$

b. $(a^2b^3)^2$

$$\frac{1}{(a^2b^3)^2} = \frac{1}{a^{2 \cdot 2} b^{3 \cdot 2}} = \frac{1}{a^4 b^6}$$

29. Marty collects stamps. Each week he is sure to add seven new stamps to his collection. (Note: Marty had 21 stamps when he first started collecting them)

Make a table representing the number of stamps Marty has each week:

week	x	0	1	2	3	4	5
Stamps	y	21	28	35	42	49	56

Write an equation to represent this information.

$$y = 7x + 21$$

$$y = 7x + 21$$

30. Find the error(s) in the solution at right. Explain what the error(s) is/are and show how to solve the equation correctly.

$$(2x + 5) - (5x + 4) = 6 - 2(x - 3)$$

$$2x + 5 - 5x + 4 = 6 - 2x - 6$$

$$-3x + 9 = -2x$$

$$9 = x$$

The errors are circled.
 They should have distributed the negative to both terms

$$\begin{array}{r} 2x + 5 - 5x - 4 = 6 - 2x + 6 \\ -3x + 1 = 12 - 2x \\ +3x \quad \quad +3x \\ \hline 1 = 12 + x \\ -12 = -12 \\ \hline -11 = x \end{array}$$

$$\boxed{x = -11}$$

32. Simplify: $\frac{15x^2y}{20xy^2}$

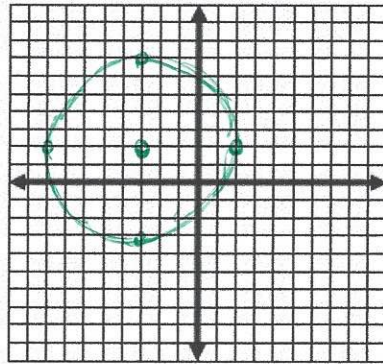
$$= \frac{3 \cancel{x} \cancel{x} y}{4 \cancel{x} \cancel{y} y} = \frac{3x}{4y}$$

33. Graph the following equation: $(x+3)^2 + (y-2)^2 = 25$

$$(x-h)^2 + (y-k)^2 = r^2$$

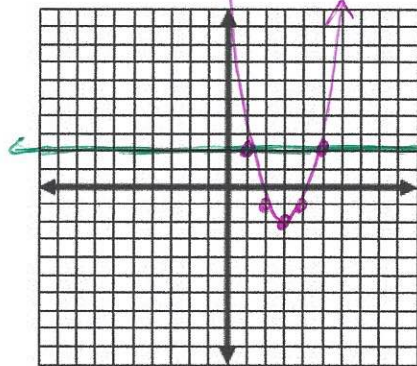
center $(h, k) = (-3, 2)$

radius = 5



34. Solve the following equation $(x-3)^2 - 2 = 2$ using two methods:

a. Graphically



POINTS OF INTERSECTION

$(1, 2)$ & $(5, 2)$

$x=1$ $x=5$

b. Algebraically

$$y = (x-3)^2 - 2$$

$$y = 2$$

$$(x-3)^2 - 2 = 2$$

$$(x-3)^2 = 4$$

$$x-3 = \pm\sqrt{4}$$

$$x-3 = \pm 2$$

$$x-3 = 2$$

$$x-3 = -2$$

$x=5$ & $x=1$

35. Solve the following equation: $\frac{x}{3} + \frac{x}{5} = \frac{8x-3}{15}$

$$\frac{15x}{3} + \frac{15x}{5} = 8x - 3$$

$$5x + 3x = 8x - 3$$

$$8x = 8x - 3$$

$$-8x \quad -8x$$

$$0x = -3$$

$$0 \neq -3$$

NO SOLUTION

36. Solve the following equation: $\sqrt{x+3} - 5 = 3$

$$(\sqrt{x+3})^2 = (8)^2$$

$$x+3 = 64$$

$$x = 61$$

$$\sqrt{61+3} - 5 = 3$$

$$\sqrt{64} - 5 = 3$$

$$8 - 5 = 3$$

$$3 = 3$$

$$\checkmark$$

37. Solve the following equation: $\sqrt{x-1} = 2x-3$. Check your answer graphically.

$$40x^2$$

$$-8x$$

$$-5x$$

$$-13x$$

$4x$	-5
$4x^2$	$-5x$
-2	$-8x$
	10

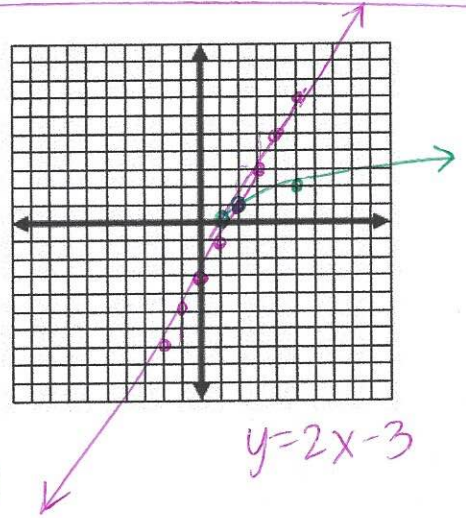
$$(\sqrt{x-1})^2 = (2x-3)^2$$

$$x-1 = (2x-3)(2x-3)$$

$$x-1 = 4x^2 - 6x - 6x + 9$$

$$x-1 = 4x^2 - 12x + 9$$

$$-x+1 \quad \quad \quad -x \quad +1$$



$$(4x-5)(x-2) = 0$$

$$4x-5=0 \quad x-2=0$$

$$x = 5/4 \quad x = 2$$

38. Bubba shoots a bottle rocket off upward to see if it will go up and over his neighbor's tree of 40 ft. He shoots the firework off at a point on the ground 20 feet from the base of the tree and is excited when it just clears the tips of the tree and then goes on to land in his neighbor's yard.

a. What type of equation models this situation?

Quadratic $y = a(x-h)^2 + k$

b. Develop an equation that models the height of the rocket

$$y = a(x-h)^2 + k$$

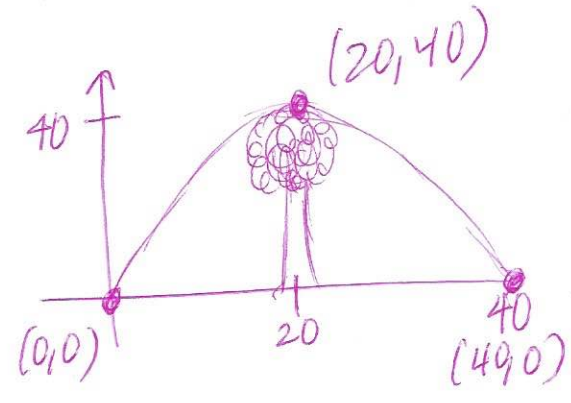
$$0 = a(0-20)^2 + 40$$

$$0 = a(-20)^2 + 40$$

$$0 = 400a + 40$$

$$\frac{-40}{400} = \frac{400a}{400}$$

$$a = -\frac{1}{10}$$



$$y = -\frac{1}{10}(x-20)^2 + 40$$